(b) $\sqrt{n} \notin \mathbb{Q}$ if $n$ is not a perfect square (HINT: write $n=k^{2} r$, where $r$ does not contain any square factor),

If $n$ is not a perfect square, then at least one of its factors is not a square. So we can write $n=k^{2} r$ where $r$ does not contain any square factors.

Now, we argue by contradiction. Suppose that $n$ is not a perfect square and $\sqrt{n} \in \mathbb{Q}$.
Then we can write $\sqrt{n}=p \mid q, p, q \in \mathbb{N}$, where $p$ and $q$ have no common factors (p/q is in its simplest form).
Then $n=p^{2} / q^{2}=k^{2} r$ or, equivalently, $r=p^{2}$. But this is imposible because $q^{2} k^{2}$
$r$ does not contain square factors. Hence, $\sqrt{n} \notin Q$

